**Solve a quartic equation**

Solve $\left(x-3\right)^{4}+\left(x-5\right)^{4}=2018$ for real and complex roots.

Note that the equation is power 4, not power 2.

Any easy way to solve?

 Put $u=x-4$, the equation becomes

 $\left(u+1\right)^{4}+\left(u-1\right)^{4}=2018$

 $2 u^{4}+ 12 u^{2}+2 =2018$

 $ u^{4}+ 6 u^{2}+1 =1009$

 $u^{4}+ 6 u^{2}-1008 =0$

 $\left(u^{2}\right)^{2}+ 6 \left(u^{2}\right)-1008 =0$

By quadratic equation formula,

 $u^{2}=-3\pm 3\sqrt{113}$

 $u=\pm \sqrt{-3\pm 3\sqrt{113}}$

 $x-4=\pm \sqrt{-3\pm 3\sqrt{113}}$

 $∴x=4\pm \sqrt{-3+3\sqrt{113}}, 4\pm i\sqrt{3+3\sqrt{113}}$

Figure out why we can use the $u=x-4$.

**Yue Kwok Choy**

**4/6/2018**